Inclusion-Exclusion Principle

COLUMBUS STATE UNIVERSITY

Math 1001

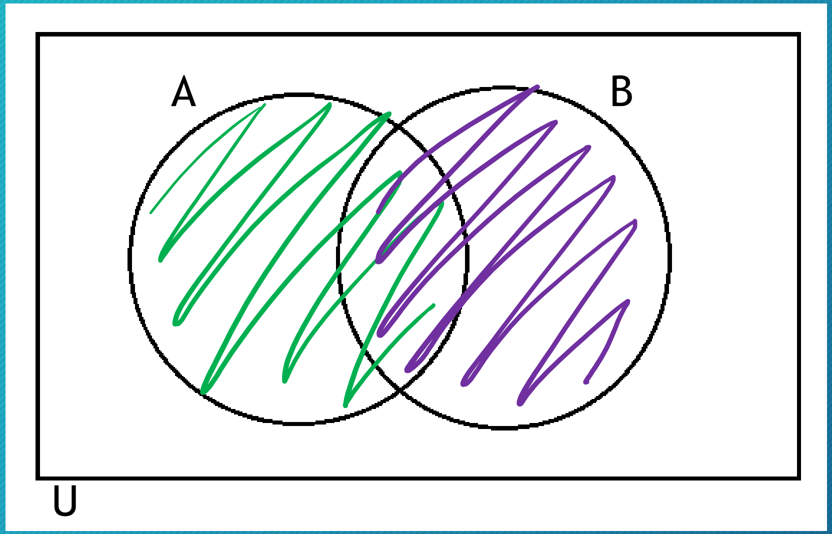
Quantitative Skills and Reasoning

The Inclusion-Exclusion Principle

cardinality

- Recall that $n(A)$ represents the number of elements in a set A.
- For all finite sets A and B,

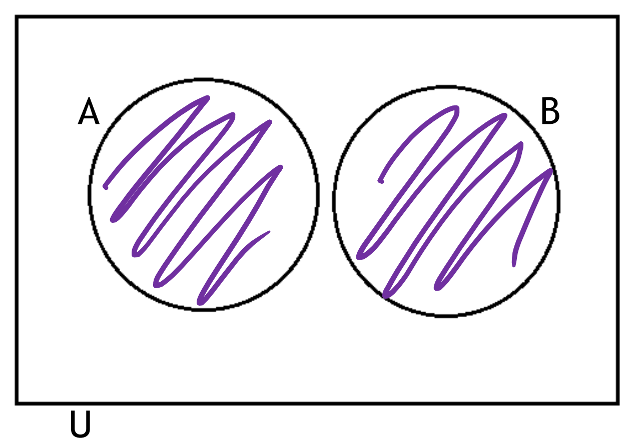
$$n(A \cup B) = n(A) + n(B) - \underbrace{n(A \cap B)}_{\text{removes double counting}}$$



The Inclusion-Exclusion Principle

- What must be true of the finite sets *A* and *B* if

$$n(A \cup B) = n(A) + n(B) \ ?$$



A and *B* must not overlap at all, thus *A* and *B* are disjoint sets.

Application of Inclusion-Exclusion

- A school finds that 860 of its students are registered in English, 530 are registered in a foreign language, and 325 are registered in both English and a foreign language. How many total students are registered in English or a foreign language?

intersection union

- Let E be the set of students registered in English
- Let F be the set of students registered in a foreign language

$$n(E \cup F) = n(E) + n(F) - n(E \cap F)$$

$$= 860 + 530 - 325$$

$$= 1065 \text{ students}$$

The Percent Inclusion-Exclusion Formula

- Let $p(A)$ represent the percent in set A. of total in Universal set
- For all finite sets A and B,

$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$

- This is essentially the same formula we've used before, but we are thinking in percentages instead of cardinality.
- It may help to think of 100 as the total in the Universal set, and each percentage is a breakdown of how many of that hundred elements.

The Percent Inclusion-Exclusion Formula

- For example, suppose a blood donation organization reports that about
  - 15% of the U.S. population is Rh-. $p(A)$
  - 16% of the U.S. population has the B antigen. $p(B)$
  - 28% of the U.S. population is Rh- or has the B antigen. $p(A \cup B)$
- Use the percent inclusion-exclusion formula to estimate the percent of the U.S. population that is Rh- and has the B antigen. $p(A \cap B)$

$$\boxed{p(A \cup B)} = p(A) + p(B) - p(A \cap B)$$

$$28\% = 15\% + 16\% - \boxed{p(A \cap B)}$$

$$28\% + p(A \cap B) = 15\% + 16\% \quad -28\%$$

$$-28\%$$

$$p(A \cap B) = 15\% + 16\% - 28\%$$

$$p(A \cap B) = 3\%$$