Inductive Reasoning

MATH 1001

Quantitative Skills and Reasoning

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Inductive Reasoning

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- **Inductive reasoning** is the process of coming to a conclusion by observing specific examples, often by locating a pattern.
  - If you examine a list of numbers and use an established pattern to predict the next number in the list, you are using inductive reasoning.
  - Use inductive reasoning to predict the next number in the following lists:

+4 +4 +4 +4 +4

2, 6, 10, 14, 18, ?

Each successive number is 4 larger than the preceding number. We predict **22** to be the next number.

+1 +2 +3 +4 +5

1, 2, 4, 7, 11, ?

The difference between the first two numbers is 1. The next two differ by 2… We predict **16** to be the next number.

Inductive Reasoning

- Consider some other examples of Inductive Reasoning:
  - My sister has had three daughters. Therefore, her next child will also be a girl.
  - The purple flowers in my yard have bloomed every other year for the past ten years. They bloomed last year, so they will not bloom this year.
  - Auburn Football has beaten Texas A&M in Texas every time they have played there this century. Therefore, the next time they play there, Auburn will win.

Conjecture

- We can also use inductive reasoning to make a *conjecture* about an arithmetic procedure.
- A conjecture is, in essence, a theory.
  - It is based on insufficient evidence.
  - We have not provided proof of correctness.
  - It could still be proven to be true or shown to be false.
  - It’s a useful step in the scientific process as well as in developing mathematical ideas, but it is incomplete.

Conjecture and Inductive Reasoning

- Consider the following procedure:
  - Pick a number
  - Subtract 3 from this number
  - Multiply this difference by 4
  - Add 12 to this product
  - Divide this sum by 2
- Take a moment to perform this process on at least three different numbers between 1 and 10.
- Examine the results. Is there a pattern between the number you start with and what you end up with that remains the same no matter where you start?

Conjecture and Inductive Reasoning

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- Here is the process as performed on each value from 1 to 10. If three examples weren't enough to establish a pattern, use these examples.
  - 1: 1-3 = -2, 4(-2) = -8, -8+12 = 4, 4/2 = 2
  - 2: 2-3 = -1, 4(-1) = -4, -4+12 = 8, 8/2 = 4
  - 3: 3-3 = 0, 4(0) = 0, 0+12 = 12, 12/2 = 6
  - 4: 4-3 = 1, 4(1) = 4, 4+12 = 16, 16/2 = 8
  - 5: 5-3 = 2, 4(2) = 8, 8+12 = 20, 20/2 = 10
  - 6: 6-3 = 3, 4(3) = 12, 12+12 = 24, 24/2 = 12
  - 7: 7-3 = 4, 4(4) = 16, 16+12 = 28, 28/2 = 14
  - 8: 8-3 = 5, 4(5) = 20, 20+12 = 32, 32/2 = 16
  - 9: 9-3 = 6, 4(6) = 24, 24+12 = 36, 36/2 = 18
  - 10: 10-3 = 7, 4(7) = 28, 28+12 = 40, 40/2 = 20

- You may notice first that when we look at it listed out this way, the results all increase by 2s.
- This is true, but not the pattern we're looking for.
- We want to know how the process changes the input (the number we picked).

Conjecture and Inductive Reasoning

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  - 4: 4-3 = 1, 4(1) = 4, 4+12 = 16, 16/2 = 8
  - 5: 5-3 = 2, 4(2) = 8, 8+12 = 20, 20/2 = 10
  - 6: 6-3 = 3, 4(3) = 12, 12+12 = 24, 24/2 = 12
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  - Instead, notice that in each of these cases, the resulting number is 2 times the original number.
  - We *conjecture* that following the given procedure produces a number that is two times the original number.

Conjecture and Inductive Reasoning

Using inductive reasoning and this procedure,

- Pick a number
- Subtract 3 from this number
- Multiply this difference by 4
- Add 12 to this product
- Divide this sum by 2

we have established a theory about what the procedure does to the input by looking for patterns.

Patterns, however, can be misleading and don't always hold up.

We need a different sort of reasoning to *prove* that this process always doubles the input, so we will come back to this example in the Deductive Reasoning video.