Logarithmic Functions

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Logarithmic Functions

- $A = 100(2^t)$
- 1600? 3200? 6400?
- to be 5000? 5001 5002

$t = ?$ $A = 5000$

$\boxed{5 < t < 6}$

$t = 3$ $A = 100 \cdot 2^3 = 800$

$t = 4$ $A = 100 \cdot 2^4 = 1600$

$t = 5$ $A = 100 \cdot 2^5 = 3200$

$t = 6$ $A = 100 \cdot 2^6 = 6400$

Logarithms

- $y = \log_b x$ is equivalent to $x = b^y$
- $b > 0, b \neq 1$

$\cancel{100}\,2^a = 5000$

$2^a = 50$

$2^a = 4 \quad a = \log_2 4 = 2$

$5 < a < 6 \Rightarrow \log_2 50 = a$

Exponential	Logarithmic
$2^a = 50$	$a = \log_2 50$
$3^x = 12$	$x = \log_3 12$
$13^t = 25$	$t = \log_{13} 25$

Logarithms

- $4^3 = 64$?

$\log_4 64$

$4^{\#} = 64$

$4^{\log_4 64} = 64$

Logarithms and Exponential Functions

- $2 = \log_{10}(x+5)$ in exponential form.

$10^2 = x+5$

- $2^{3x} = 64$ in logarithmic form.

$\log_2 64 = 3x$

Evaluate Logarithmic Expressions

- $\log_4 64 = a \Rightarrow 4^a = 64 \Rightarrow a = 3 \Rightarrow \log_4 64 = 3$
- $\log_2\left(\frac{1}{16}\right) = -4$

$$\boxed{2^a} = \frac{1}{16} = \left(\frac{1}{2}\right)^4 = \boxed{2^{-4}}$$

Solve a Logarithmic Equation

- Solve: $\log_2 x = 4$ $\Rightarrow$ $2^4 = x$

$16 = x$

Common and Natural Logarithms

- $f(x) = \log_{10} x$: **common logarithmic function**: $\boxed{f(x) = \log x}$.
- $f(x) = \log_e x$: **natural logarithmic function**: $f(x) = \ln x$.
- Examples:

$$\ln x = \log_e x$$

$$\boxed{\log 100 = 2}$$

$$\log_{10} 100 = x = 2$$

$$10^x = 100$$

$$x = 2$$

and

$$\ln e^2 = 2$$

$$\boxed{\ln e^2} = \log_e e^2 = x = 2$$

$$e^x = e^2 \Rightarrow x = 2$$

Solve Common and Natural Log Equations

- Solve each of the following equations. Round to the nearest thousandth.
  - $\log x = -2.5$ $\Rightarrow$ $\log x = \boxed{\log_{10} x = -2.5}$

    $\boxed{10^{-2.5} = x}$
  - $\ln x = 5$

    $e^5 = x$

Other Logarithmic Graphs (1,0)

$y = \log_b x$

- The base: