Conditional and Biconditional Statements

Math 1001

Quantitative Skills and Reasoning

Truth Table Involving a Conditional

- Construct a truth table for $[\sim p \vee (\sim q \vee p)] \rightarrow \sim p$.

p	*q*	[~*p*	v	(~*q*	v	*p*)]	→	~*p*
T	T	F	T	F	T	T	F	F
T	F	F	T	T	T	T	F	F
F	T	T	T	F	F	F	T	T
F	F	T	T	T	T	F	T	T

An Equivalent Form of the Conditional

disjunctive form of the conditional

- Show that $p \rightarrow q \equiv \sim p \vee q$ using truth tables.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\sim p$	$\vee$	q
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	F

$\equiv$

logically equivalent

Rewriting a Conditional Statement

$p \rightarrow q \equiv \sim p \vee q$

- Write each of the following in its equivalent disjunctive form:
  - If I could catch a ball, I would join the softball team.

    I can't play catch a ball or I would join the softball team.
  - If I don't drink coffee, then I will get a headache.

    I drink coffee or I will get a headache.

The Negation of a Conditional

- Because $p \rightarrow q \equiv \sim p \vee q$, an equivalent form of $\sim(p \rightarrow q)$ is given by $\sim(\sim p \vee q)$, which, by one of De Morgan's laws, can be expressed as the conjunction $p \wedge \sim q$.

- Thus, $\sim(p \rightarrow q) \equiv \sim(\sim p \vee q) \equiv p \wedge \sim q$, or more simply,
  - $\sim(p \rightarrow q) \equiv p \wedge \sim q$

conjunctive form

Write the Negation of a Conditional Statement

$\sim(p \rightarrow q) \equiv p \wedge \sim q$

- Write the negation of each conditional statement:
  - If [I complete the activities], then [I will pass the class].

    I complete the activities (and) I don't pass the class.

  - If [the guest speaker is late], then [we will replace him].

    The guest speaker is late but we won't replace him.

The Biconditional

- The statement $(p \rightarrow q) \wedge (q \rightarrow p)$ is called a **biconditional** and is denoted by $p \leftrightarrow q$, which is read as "p if and only if q."
- $p \leftrightarrow q \equiv [(p \rightarrow q) \wedge (q \rightarrow p)]$
- A truth table for $p \leftrightarrow q$:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

- Note that this truth table shows that a biconditional statement is true only when p and q have the *same* truth value.

Determine the Truth Value of a Biconditional

- State whether each biconditional statement is true or false:
  - $x - 7 = 3$ if and only if $x = 10$

    Both equations are true when $x = 10$, and both are false when $x \neq 10$. Both equations have the same truth value for any value of x, so this is a true statement.
  - $x^2 = 49$ if and only if $x = 7$.

    Both equations are true when $x = 7$, but, the first equation is also true when $x = -7$, and the second equation is false. Therefore, this is a false statement.