THE COUNTING PRINCIPLE

Math 1001

Quantitative Skills and Reasoning

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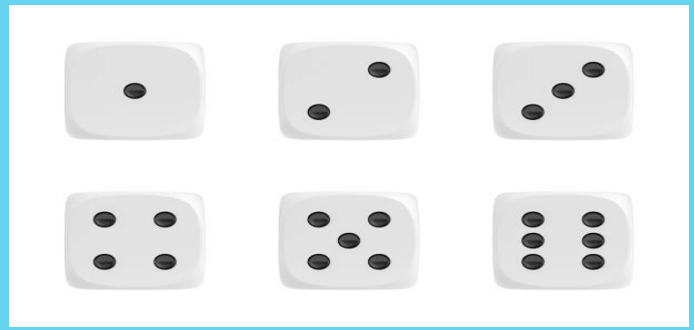
- **Combinatorics** is the study of counting the different outcomes of some task.
- For example, if a coin is flipped, the side facing upward will be a head or a tail.
- If a regular six-sided die is rolled, the possible outcomes are:



- The outcomes can also be listed as {1, 2, 3, 4, 5, 6}.
- There are six possible outcomes.

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COUNTING BY MAKING A LIST

- List and then count the number of different outcomes that are possible when one letter from the word *Banana* is chosen.

- The possible outcomes are {B, a, n}.
- There are three possible outcomes.

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COMBINATORICS TERMINOLOGY

- In combinatorics, an **experiment** is an activity with an observable outcome.
  - Flipping a coin
  - Rolling a die
  - Choosing a letter from the word Banana
- The set of all possible outcomes of an experiment is called the **sample space** of the experiment.
  - {head, tail} or {H, T}
  - {1, 2, 3, 4, 5, 6}
  - {B, a, n}

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COMBINATORICS TERMINOLOGY

- An **event** is one or more of the possible outcomes of an experiment.
  - Flipping a coin and having a head show on the upward face
  - Rolling a 5 when a die is tossed
  - Choosing a 'n' from one of the letters in the word Banana
- An event is a *subset* of the sample space.

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LISTING THE ELEMENTS OF AN EVENT

- One number is chosen from the sample space

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$

- List the elements in the following events:
  - The number is even.
    - {2, 4, 6, 8, 10, 12, 14, 16, 18}
  - The number is divisible by 5.
    - {5, 10, 15}
  - The number is a prime number.
    - {2, 3, 5, 7, 11, 13, 17}

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COUNTING BY MAKING A TABLE

Each of the experiments given above illustrates a *single-stage experiment*.

A **single-stage experiment** is an experiment for which there is a single outcome.

Some experiments have two, three, or more stages.

Such experiments are called **multi-stage experiments**.

To count the number of outcomes of such an experiment, a systematic procedure is helpful.

Using a table to record results is one such procedure.

COUNTING

- Consider the two-stage experiment of rolling two dice, one red and one green.
- How many different outcomes are possible?
- To determine the number of outcomes, make a table with the different outcomes of rolling the green die across the top and the different outcomes of rolling the red die down the side.

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

COUNTING BY MAKING A TABLE

- By counting the number of entries in the diagram, we see that there are 36 different outcomes of the experiment of rolling two dice.

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

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Counting by Making a Table

- From the table, several different events can be discussed.
  - The sum of the pips (dots) on the upward face is 7.
  - There are six outcomes

{(6,1), (5,2), (4,3), (3,4), (2,5), (1,6)}

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Counting by Making a Table

- From the table, several different events can be discussed.
  - The sum of the pips (dots) on the upward face is 11.
  - There are two outcomes

    {(6,5), (5,6)}

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Counting by Making a Table

- From the table, several different events can be discussed.
  - The number of pips on the upward faces are equal.
  - There are six outcomes

{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

COUNTING BY MAKING A TABLE

- Two-digit numbers are formed from the digits 1, 3, and 6. Find the sample space and determine the number of elements in the sample space.

- Use a table to list all the different two-digit numbers that can be formed by using the digits 1, 3, and 6.

	1	3	6
1	11	13	16
3	31	33	36
6	61	63	66

COUNTING BY MAKING A TABLE

- The sample space is:

{11, 13, 16, 31, 33, 36, 61, 63, 66}

- The number of elements in the sample space is:

There are 9 elements in the sample space.

	1	3	6
1	11	13	16
3	31	33	36
6	61	63	66

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