COMPOUND INTEREST

Math 1001

Quantitative Skills and Reasoning

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COMPOUND INTEREST

- Simple interest is generally used for loans of 1 year or less.
- For loans of more than 1 year, the interest paid on the money borrowed is called *compound interest*.
- Compound interest is interest calculated not only on the original principal, but also on any interest that has already been earned.

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COMPOUND INTEREST

- To illustrate compound interest, suppose you deposit $100 in a savings account earning 5% interest, compounding annually (once a year).
- During the first year, the interest earned is calculated as follows:

$I = Prt$

$I = 100(0.05)(1) = \$5$

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COMPOUND INTEREST

- At the end of the first year, the total amount in the account is

$$A = P + I$$

$$A = 100 + 5 = \$105$$

- During the second year, the interest earned is calculated using the amount in the account at the end of the first year.

$$I = Prt$$

$$I = 105(0.05)(1) = \$5.25$$

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COMPOUND INTEREST

- Note that the interest earned during the second year ($5.25) is greater than the interest earned during the first year ($5).
- This is because the interest earned during the first year was added to the original principal, and the interest for the second year was calculated using this sum.
- If the account earned simple interest rather than compound interest, the interest earned each year would be the same ($5).

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COMPOUND INTEREST

- At the end of the second year, the total amount in the account is

$$A = P + I$$

$$A = 105 + 5.25 = \$110.25$$

- The interest earned during the third year is calculated using the amount in the account at the end of the second year.

$$I = Prt$$

$$I = 110.25(0.05)(1) = \$5.51$$

COMPOUND INTEREST

- The interest earned each year keeps increasing.

  This is the effect of compound interest.

- In the previous example, the interest is compounded annually.
- However, compound interest can be compounded:
  - Semiannually (twice a year)
  - Quarterly (four times a year)
  - Monthly
  - Daily

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COMPOUND INTEREST

- The frequency with which the interest is compounded is called the **compounding period**.
- Suppose that instead of interest compounding annually in the last example, it compounded quarterly.
- In this case, *t* is no longer 1, it's $\frac{1}{4}$ (a *quarter* of a year).

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COMPOUND INTEREST

- End of first quarter:
  - $I = Prt = 100(0.05)(1/4) = \$1.25$
  - $A = P + I = 100 + 1.25 = \$101.25$
    - End of second quarter:
      - $I = Prt = 101.25(0.05)(1/4) = \$1.265625 \approx \$1.27$
      - $A = P + I = 101.25 + 1.27 = \$102.52$
    - End of third quarter:
      - $I = Prt = 102.52(0.05)(1/4) = \$1.28$
      - $A = P + I = 102.52 + 1.28 = \$103.80$
    - End of fourth quarter:
      - $I = Prt = 103.80(0.05)(1/4) = \$1.30$
      - $A = P + I = 103.80 + 1.30 = \$105.10$

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COMPOUND INTEREST

- The total amount in the account at the end of the first year is $105.10.
- Note that this is slightly higher than the amount in the account after the first year when interest was compounded annually ($105.00)

- **In general, an increase in the number of compounding periods results in an increase in the interest earned by an account.**

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COMPOUND INTEREST

- In the previous example, we used the formulas $I = Prt$ and $A = P + I$ to show the amount of interest added to the account each quarter.
- To simplify things, we can use the formula $A = P(1 + rt)$ instead.

CALCULATE THE FUTURE VALUE

- You deposit $500 in an account earning 4% interest, compounded semiannually.

  How much is in the account at the end of 1 year?

- Interest is compounded semiannually, which is every six months. Thus, $t = \frac{6}{12} = \frac{1}{2}$ for the first six months.
- $A = P(1 + rt)$
- $A = 500\left[1 + 0.04\left(\frac{1}{2}\right)\right] = 510$

  The account holds $510 after six months.

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CALCULATE THE FUTURE VALUE

- Calculate the amount after the second six months:
- $A = P(1 + rt)$
- $A = 510\left[1 + 0.04\left(\frac{1}{2}\right)\right] = 520.20$

The account holds $520.20 after the second six months, i.e., one year.

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COMPOUND AMOUNT

- In calculations that involve compound interest, the sum of the principal and the interest that has been added to it is called the **compound amount**.
- In the previous example, the compound amount is $520.20.
- The calculation necessary to determine compound interest and compound amounts can be simplified using a formula.

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COMPOUND AMOUNT

- Consider an amount P deposited into an account paying an annual interest rate r, compounded annually.
- The interest earned during the first year is
  - $I = Prt$
  - $I = Pr(1)$ since $t = 1$
  - $I = Pr$

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COMPOUND AMOUNT

- The compound amount A in the account after 1 year is the sum of the original principal and the interest earned during the first year:
  - $A = P + I$
  - $A = P + Pr$
  - $A = P(1+ r)$ (factor P from each term)

COMPOUND AMOUNT

- During the second year, the interest is calculated on the compound amount at the end of the first year: $P(1 + r)$.
  - $I = Prt$
  - $I = P(1 + r)r(1)$ (replace P with $P(1+r)$ and $t=1$)
  - $I = P(1+ r)r$ (factor P from each term)

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COMPOUND AMOUNT

- The compound amount A in the account after two years is the sum of the compound amount at the end of the first year and the interest earned during the second year:
  - $A = P + I$
  - $A = P(1+r) + P(1+r)r$
  - $A = P(1+r)(1+r)$ (factor $P(1+r)$ from each term)
  - $A = P(1+r)^2$

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COMPOUND AMOUNT

- During the third year, the interest is calculated on the compound amount at the end of the second year: $P(1 + r)^2$.
  - $I = Prt$
  - $I = P(1 + r)^2 r(1)$ (replace P with $P(1+r)^2$ and $t=1$)
  - $I = P(1+ r)^2 r$ (factor P from each term)

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COMPOUND AMOUNT

- The compound amount A in the account after three years is the sum of the compound amount at the end of the first year and the interest earned during the second year:
  - $A = P + I$
  - $A = P(1+r)^2 + P(1+r)^2 r$
  - $A = P(1+r)^2(1+r)$ (factor $P(1+r)^2$ from each term)
  - $A = P(1+r)^3$

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- Note that the compound amount at the end of each year is the previous year's compound amount multiplied by $(1 + r)$.
- The exponent on $(1 + r)$ is equal to the number of compounding periods.
- Generalizing from this, we can state that the compound amount after n years is $A = P(1+r)^n$.

- In deriving this equation, interest was compounded annually; therefore, $t = 1$.

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