Complements and Subsets

Math 1001

Quantitative Skills and Reasoning

The Universal Set

- It is often important to be aware of the set of all elements that are under consideration.
  - For instance, when I want to schedule an appointment, there are 365 dates (in a non-leap year) available for me to choose from. March 33, for instance, is not a date under consideration.
  - When you mail a letter within the US, you know that the zip code is given by a natural number with five digits. The rational number 1/4 is not a consideration.
- The set of elements that are being considered is called the **universal set**. We will use the letter U to denote the universal set.

The Complement of a Set

- The **complement** of a set *A*, denoted by *A'*, is the set of all elements of the universal set *U* that are not elements of *A*.
  - Let *U* = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, *S* = {2, 3, 6, 7} and *T* = {*x* | *x* < 8 and *x* ∈ the odd Natural numbers}. Find:
    - *S'*

      T = {1, 3, 5, 7}

      = {0, 1, 4, 5, 8, 9}
    - *T'*

      = {0, 2, 4, 6, 8, 9}

Complements

- Because the universal set contains all elements under consideration, the complement of the universal set is the empty set.
  - $U' = \emptyset$
- Conversely, because the empty set has no elements and the universal set contains all the elements under consideration, the complement of the empty set is the universal set.
  - $\emptyset' = U$

Subsets

$\leq$

- Set A is a **subset** of set B, denoted by $A \subseteq B$, if and only if every element of A is also an element of B.
  - Consider the set of students attending Columbus State University and the set of students in this class. Every student in this class is a student attending Columbus State University. The set of students in this class is a subset of the set of students attending CSU.

Subsets

- Here are two fundamental relationships:
  - $A \subseteq A$, for any set A
  - $\emptyset \subseteq A$, for any set A
- To convince yourself of this, consider the following:
  - Because every element in the arbitrary set A is an element of the arbitrary set A, we know that $A \subseteq A$.
  - Because every element in the empty set (there are none) is an element of an arbitrary set A, we know that $\emptyset \subseteq A$.

Subsets

- The notation $A \not\subseteq B$ is used to denote that A is *not* a subset of B.
- To show that A is not a subset of B, it is necessary to find at least one element of A that is not an element of B.
- Determine whether each statement is true or false:
  - $\{4, 8, 12, 16\} \subseteq \{0, 4, 8, 12, 16, 20, 24\}$ TRUE
  - $\{0, 1, 2 \ldots\}$
  - $\boldsymbol{W} \subseteq \boldsymbol{N}$ FALSE
  - $\{1, 2, 3, \ldots\}$ $0 \in W$ but $0 \notin N$
  - $\{1, 3, 5\} \subseteq \{1, 3, 5\}$ TRUE
  - $A \subseteq A$
  - $\emptyset \subseteq \{a, b, c\}$ TRUE
  - $\emptyset \subseteq A$

All Subsets of a Set

- Let *C* be the four toppings that a bakery stand offers on its sweet buns.
  - *C* = {sprinkles, fruit spread, chocolate, cream cheese frosting}
- List all the subsets of *C*.
  - Subsets with 0 elements: { }
  - Subsets with 1 element: {sprinkles}, {fruit spread}, {chocolate}, {cream cheese frosting}
  - Subsets with 2 elements: {sprinkles, fruit spread}, {sprinkles, chocolate}, {sprinkles, cream cheese frosting}, {fruit spread, chocolate}, {fruit spread, cream cheese frosting}, {chocolate, cream cheese frosting}
  - Subsets with 3 elements: {sprinkles, fruit spread, chocolate}, {sprinkles, chocolate, cream cheese frosting} {sprinkles, fruit spread, cream cheese frosting}, {fruit spread, chocolate, cream cheese frosting}
  - Subsets with 4 elements: {sprinkles, fruit spread, chocolate, cream cheese frosting}

Number of Subsets of a Set

- Notice from the last example, our set *C* had 4 elements. In total, it had 16 subsets.
- Suppose we have a set with *n* elements, where *n* is a natural number. This set has 2^n subsets.
  - For example,
    - {1, 2, 3, 4, 5, 6} has 6 elements and thus has $2^6 = 64$ subsets.
    - {basil, sage, parsley, rosemary, thyme, dill, cilantro, fennel, lemongrass, mint, tarragon, chives, bay leaf} has 13 elements and thus $2^{13} = 8192$ subsets.
    - $\emptyset$ has 0 elements and thus $2^0 = 1$ subset (namely, itself).