

Quantifiers and Negation

Math 1001

Quantitative Skills and Reasoning



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Quantifiers and Negation

- In a statement, the word *some* and the phrases *there exist* and *at least one* are called **existential quantifiers**.
 - These quantifiers are used to assert the existence of something.
- In a statement, the words *none*, *no*, *all*, and *every* are called **universal quantifiers**.
 - The universal quantifiers *no* and *none* deny the existence of something.
 - The universal quantifiers *all* and *every* assert that every element of a given set satisfies some condition.

Quantifiers and Negation

- Recall that the negation of a false statement is a true statement, and vice versa.
- Consider the statement “All dogs bite,” which is false.
 - The negation of this statement might be thought to be “No dogs bite.” But this statement is also false, so it cannot be the correct negation.
 - Instead, the negation is that “*Some* dogs do not bite.”
 - Also stated as “There exists a dog that is does not bite.”
 - Or, “At least one dog does not bite.”

Quantifiers and Negation

- How to write the negation of some quantified statements:

Statement	Negation
All X are Y .	Some X are not Y .
Some X are Y .	All X are not Y .

- If the Universal quantifier is *no* or *none*, rewrite the statement so that it begins with *all* or *every* so that you can apply the rules above more directly.

Negation of a Quantified Statement

- Write the negation of each of the following statements:
 - All teachers write legibly.
Some teachers do not write legibly.
 - Some restaurants are open.
All restaurants are closed.
 - All concerts are worth the price of admission.
Some concerts are not worth the price of admission.
 - ~~No~~^{All} even numbers are ~~divisible~~^{not} by 3.
[There exists_{some}] an even number that is divisible by 3.