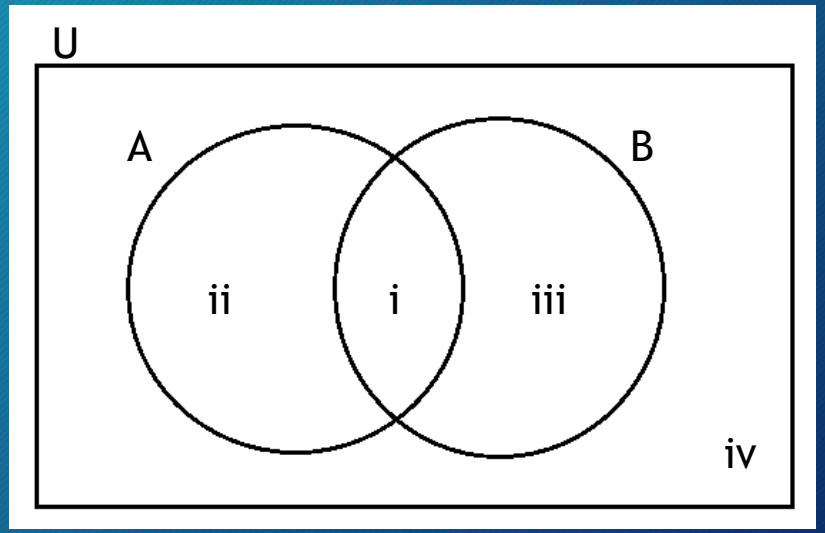
Showing Equality using Venn Diagrams (2 Sets)

Math 1001

Quantitative Skills and Reasoning

Venn Diagrams and Equality of Sets

- The given Venn diagram shows the four regions formed by two intersecting sets in a universal set *U*.
- It shows the four possible relationships that can exist between an element of a universal set *U* and two sets *A* and *B*.
- An element of U:
  - May be an element of both *A* and *B* (Region i)
  - May be an element of *A*, but not *B* (Region ii)
  - May be an element of *B*, but not *A* (Region iii)
  - May not be an element of either *A* or *B* (Region iv)



Venn Diagrams and Equality of Sets

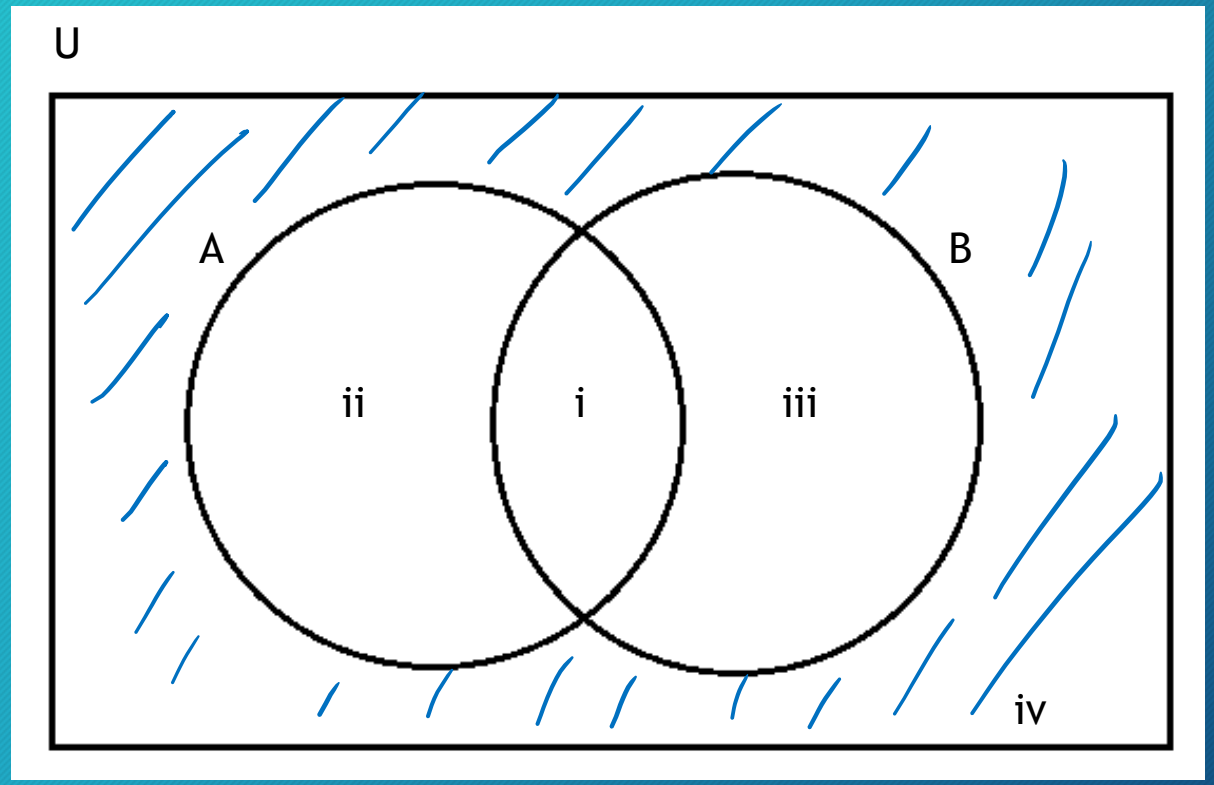
- We can use this figure to determine whether two expressions that involve two sets are equal.
- For instance, to determine whether $(A \cup B)'$ and $A' \cap B'$ are equal for all sets A and B, we find what region or regions each of the expressions represents in the figure.
  - If both expressions are represented by the same region(s), then the expressions are equal for all sets A and B.
  - If both expressions are *not* represented by the same region(s), then the expressions are *not* equal for all sets A and B.

Venn Diagrams and Equality of Sets

- Determine whether $(A \cup B)'$ and $A' \cap B'$ are equal for all sets A and B.
  - First, determine which regions are included in $A \cup B$.
  - Then, determine which regions are included in $(A \cup B)'$.

Venn Diagrams and Equality of Sets

- Show which regions are represented by $(A \cup B)'$.



$A \cup B$: i, ii, iii

$(A \cup B)'$: iv

Venn Diagrams and Equality of Sets

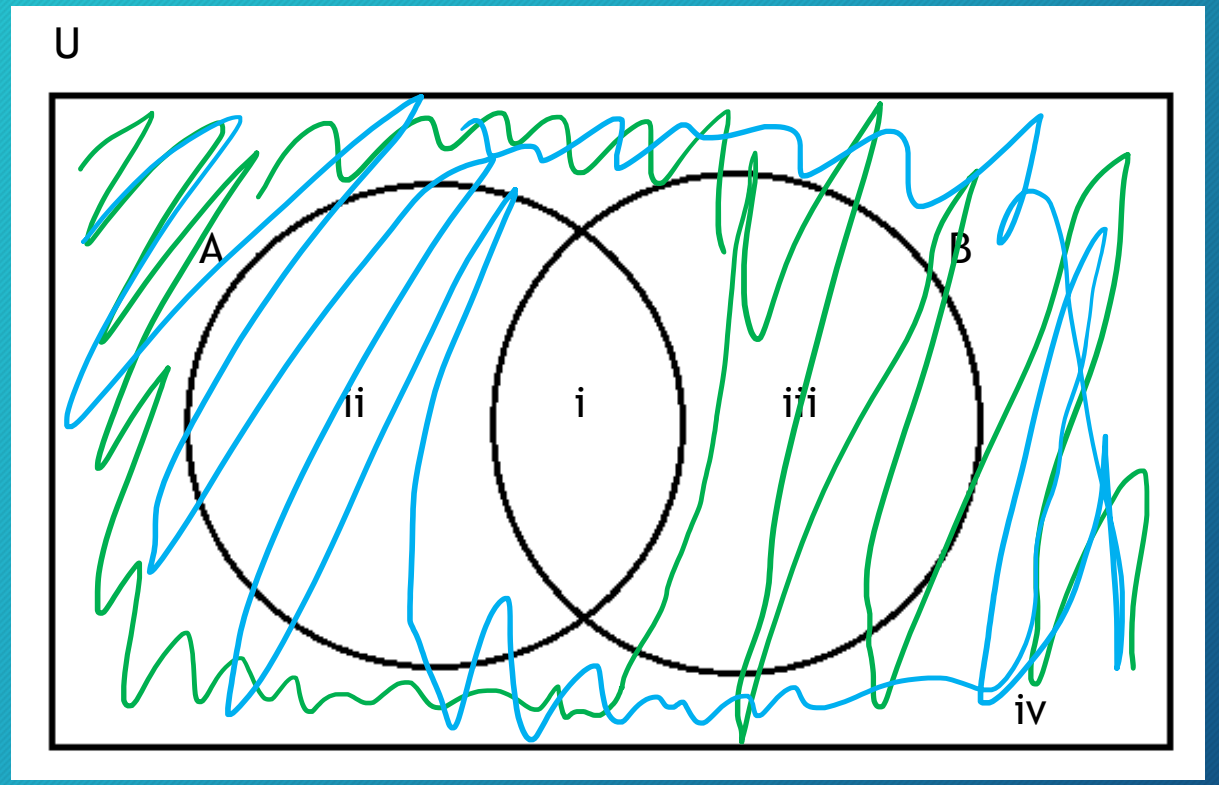
- Determine whether $(A \cup B)'$ and $A' \cap B'$ are equal for all sets A and B.
  - First, determine which regions are included in $A \cup B$.
    - $A \cup B$ includes all elements that are in set A or in set B, or in both. This includes regions i, ii, and iii.
  - Then, determine which regions are included in $(A \cup B)'$.
    - $(A \cup B)'$ includes all elements that are in neither set A nor in set B. This includes only region iv.

Venn Diagrams and Equality of Sets

- Now we must check to see if $A' \cap B'$ is also represented only by region iv.
  - First, determine which regions are included in A'.
  - Then, determine which regions are included in B'.
  - Finally, which region(s) representing the intersection of A' and B'.

Venn Diagrams and Equality of Sets

- Now we must check to see if $A' \cap B'$ is also represented only by region iv.



A′ : iii , iv

B′ : ii , iv

A′ ∩ B′ : iv

Venn Diagrams and Equality of Sets

- Now we must check to see if $A' \cap B'$ is also represented only by region iv.
  - First, determine which regions are included in A'.
    - As A includes regions i and ii, we know that A' is represented by the other regions, iii and iv.
  - Then, determine which regions are included in B'.
    - As B includes regions i and iii, we know that B' is represented by the other regions, ii and iv.
  - Finally, which region(s) representing the intersection of A' and B'.
    - The region common to both A' and B' is region iv.
- Since both are represented only by region iv, we know that $(A \cup B)' = A' \cap B'$ for all sets A and B.

De Morgan's Laws

- For all sets A and B,

$$(A \cup B)' = A' \cap B'$$

and

$$(A \cap B)' = A' \cup B'$$

- The first of the two laws is stated, "The complement of the union of two sets is equal to the intersection of the complements of the sets."
- The second is stated, "The complement of the intersection of two sets is equal to the union of the complements of the sets.