Showing Equality using Venn Diagrams (3 Sets)

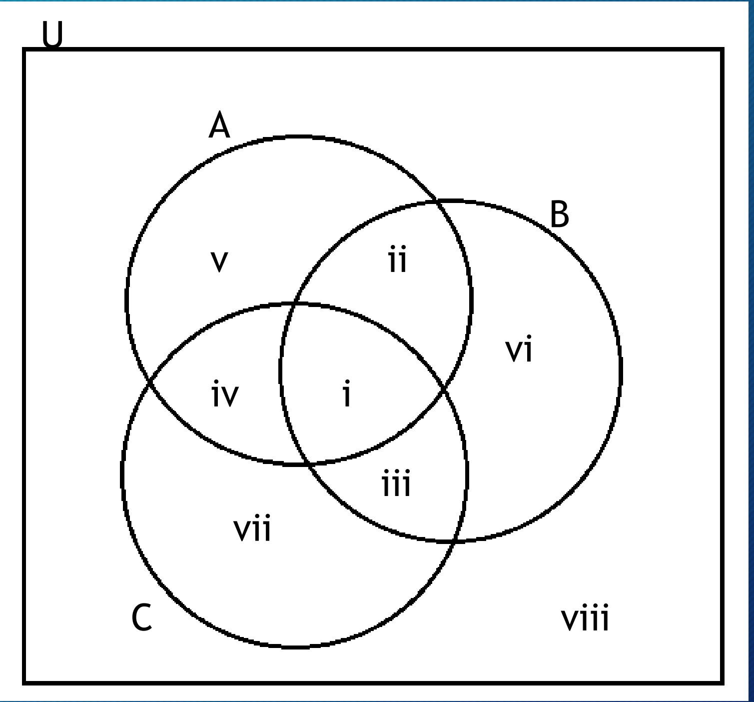
Math 1001

Quantitative Skills and Reasoning

COLUMBUS STATE UNIVERSITY

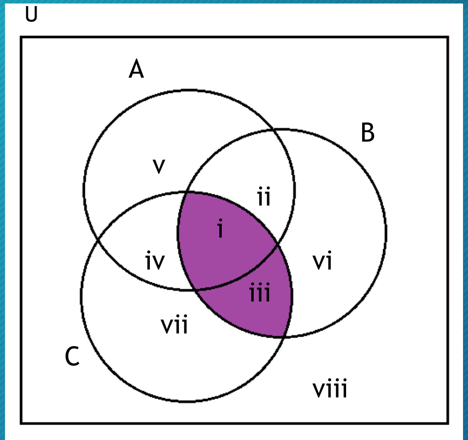
Venn Diagrams Involving Three Sets

- The given Venn diagram shows the eight regions formed by three intersecting sets in a universal set *U*.
- It shows the eight possible relationships that can exist between an element of a universal set *U* and three sets *A*, *B* and *C*.
- An element of U:
  - May be an element of each *A*, *B* and *C* (Region i)
  - May be an element of *A* and *B*, but not *C* (Region ii)
  - May be an element of *B* and *C*, but not *A* (Region iii)
  - May be an element of *A* and *C*, but not *B* (Region iv)
  - May be an element of *A*, but not *B* or *C* (Region v)
  - May be an element of *B*, but not *A* or *C* (Region vi)
  - May be an element of *C*, but not *A* or *B* (Region vii)
  - May not be an element of *A*, *B*, or *C* (Region viii)

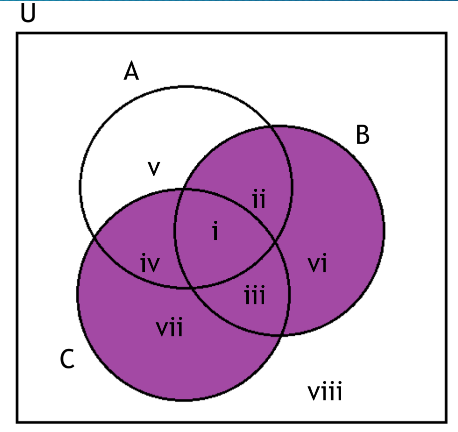


Venn Diagrams Involving Three Sets

- Which regions represent $B \cap C$?
  - $B \cap C$ is represented by all the regions common to circles B and C. Thus $B \cap C$ is represented by regions i and iii.

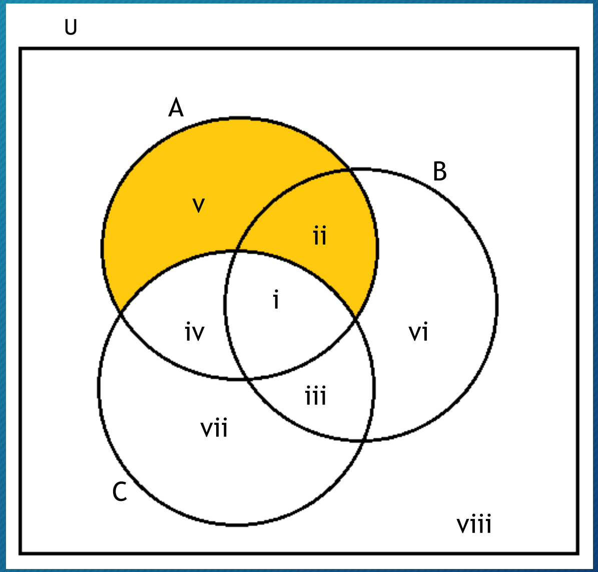


- Which regions represent $B \cup C$?
  - $B \cup C$ is obtained by joining the circles B and C. Thus $B \cap C$ is represented by regions i, ii, iii, iv, vi, and vii.



Venn Diagrams Involving Three Sets

- Which regions represent $A \cap C'$?
  - $A \cap C'$ is represented by all the regions common to circles A and simultaneously *not* in circle C. That includes the regions ii and v.

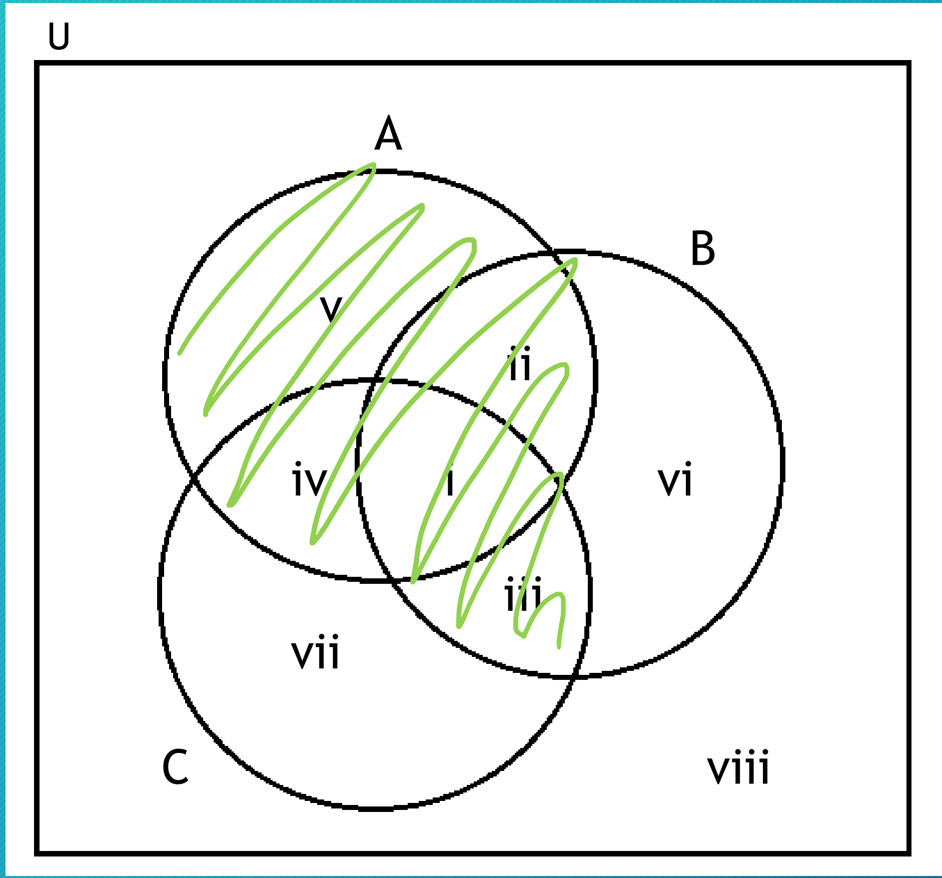


Equality of Sets

- Determine whether $A \cup (B \cap C) = (A \cup B) \cap C$ for all sets A, B, and C.
  - First, determine which regions are included in A.
  - Next, determine which regions are included in $B \cap C$.
  - Then, determine which regions are included in $A \cup (B \cap C)$.

Equality of Sets

- Determine regions representing $A \cup (B \cap C)$



A: i, ii, iv, v

B ∩ C: i, iii

A ∪ (B ∩ C):

i, ii, iii, iv, v

Equality of Sets

- Determine whether $A \cup (B \cap C) = (A \cup B) \cap C$ for all sets A, B, and C.
  - First, determine which regions are included in A.
    - A includes the regions i, ii, iv, and v.
  - Next, determine which regions are included in $B \cap C$.
    - $B \cap C$ includes all elements that are common to both B and C. These are the regions i and iii.
  - Then, determine which regions are included in $A \cup (B \cap C)$.
    - Here we include each region mentioned previously. Therefore, we have regions i, ii, iii, iv, and v.

Equality of Sets

- Now we must check to see if $(A \cup B) \cap C$ is also represented by regions i, ii, iii, iv, and v.
  - First, determine which regions are included in $A \cup B$.
  - Then, determine which regions are included in C.
  - Finally, which region(s) represent the intersection of $A \cup B$ and C.

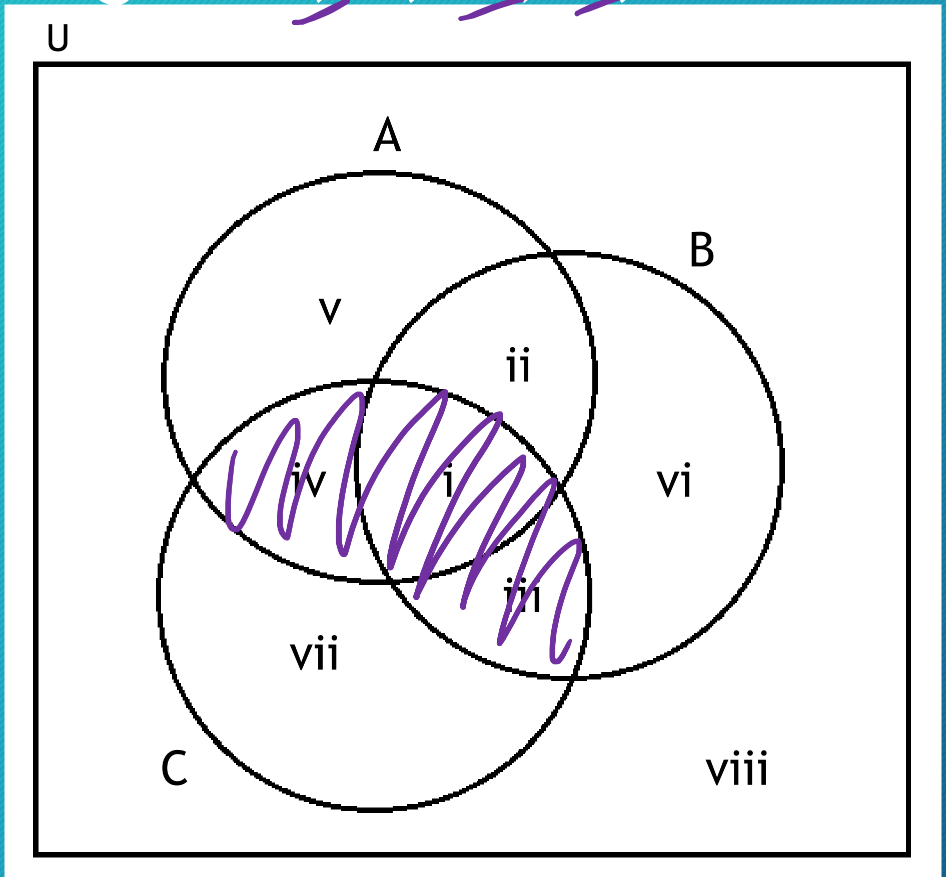
Equality of Sets

- Now we must check to see if $(A \cup B) \cap C$ is also represented by regions i, ii, iii, iv, and v.

$A \cup B$: i, ii, iii, iv, v, vi

C: i, iii, iv, vii

$(A \cup B) \cap C$: i, iii, iv



Equality of Sets

- Now we must check to see if $(A \cup B) \cap C$ is also represented by regions i, ii, iii, iv, and v.
  - First, determine which regions are included in $A \cup B$.
    - Regions include all those in the circles A and B: regions i, ii, iii, iv, v, and vi.
  - Then, determine which regions are included in C.
    - C includes regions i, iii, iv, and vii.
  - Finally, which region(s) represent the intersection of $A \cup B$ and C.
    - The regions common to both $A \cup B$ and C are region i, iii, and iv.
- Since both are represented by different regions, we know that $A \cup (B \cap C) \neq (A \cup B) \cap C$ for all sets A, B and C.

Properties of Sets

- For all sets A and B:
  - **Commutative Properties**
    - $A \cap B = B \cap A$
    - $A \cup B = B \cup A$
- For all sets A, B, and C:
  - **Associative Properties**
    - $(A \cap B) \cap C = A \cap (B \cap C)$
    - $(A \cup B) \cup C = A \cup (B \cup C)$

- Does $(B \cup C) \cap A = (A \cap B) \cup (A \cap C)$?

Yes. The commutative property of intersection allows us to write the distributive property in this way as well as the way it is written below.

$3(x-2) = 3x-6$

- **Distributive Properties**
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
  - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$