

# Tautologies and Self-Contradictions



Math 1001

Quantitative Skills and Reasoning

# Tautologies and Self-Contradictions

- A **tautology** is a statement that is always true.
- A **self-contradiction** is a statement that is always false.

# Verify a Tautology

- Show that  $p \vee (\sim p \vee q)$  is a tautology.
- Construct a truth table.

$p$	$q$	1 $p$	2 $\vee$	<del>3</del> $(\sim p$	4 $\vee$	<del>5</del> $q)$
T	T	T	T	F	T	T
T	F	T	T	F	F	F
F	T	F	T	T	T	T
F	F	F	T	T	T	F

# Verify a Self-Contradiction

- Show that  $p \wedge (\sim p \wedge q)$  is a self-contradiction.
- Construct a truth table.

$p$	$q$	1	2	<del>3</del>	4	<del>5</del>
$p$	$q$	$p$	$\wedge$	$(\sim p$	$\wedge$	$q)$
T	T	T	F	F	F	T
T	F	T	F	F	F	F
F	T	F	F	T	T	T
F	F	F	F	T	F	F

# Identify Tautologies and Self-Contradictions

- Is the statement “It is raining and it is not raining” a tautology or self-contradiction?

Self-contradiction. Both things cannot be true at the same time, so this is always false.

- Is the statement “I’m wearing socks or I’m not wearing socks” a tautology or self-contradiction?

Tautology. Because one of those statements is always true, this will always be true.

- Is the statement  $x + 3 = 9$  a tautology or self-contradiction?

Neither. The statement is not true for all values of  $x$ , and it is not false for all values of  $x$ . This is called an open statement.